

Do frictionless models of money and the price level make sense?*

Colin Rogers
Economics
University of Adelaide

Abstract:

No. As well-specified Walrasian general equilibrium systems, frictionless models are isomorphic with the Arrow-Debreu (A-D) world. It is well known that the A-D world has no role for money, credit or banks. Grafting a role for money onto a frictionless model by appending a quantity equation or cash-in-advance constraint makes the error of converting money into a friction. Furthermore, as frictionless models have no use for money or nominal values it makes no sense to use them to adjudicate between theories of the price level or to claim that they provide the theoretical foundations for monetary policy.

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*Colin Rogers
Economics
University of Adelaide*

In recent years the application of *frictionless* or *cashless* models to the analysis of monetary and fiscal theory has received considerable attention in the literature. Leading exponents of the application of such models are Woodford (2003) and Cochrane (1998, 2005). Such frictionless or cashless models are thought to be good approximations to electronic money systems, as they now exist, with final settlement of transactions through the central bank (Woodford 2003, p. 31, Cochrane 2005, p.505). Green (2005, p. 31) sees this as an important practical implication of frictionless models:

‘From the perspective of central bank economists, it is of great value to have a family of tractable models that yield intuitively appealing policy alternatives as optima.’

However, the use of frictionless models has polarised opinion among theorists. On the one hand, on the fly of his book, Woodford receives glowing praise from a distinguished list of economists. On the other hand, Willem Buiter (1999, p. 1, emphasis added) presents a strident criticism when he states:

‘It is not common for an entire scholarly literature to be based on a fallacy, that is ‘on faulty reasoning; misleading or unsound argument’. The recently revived ‘fiscal theory of the price level’ is an example of a research programme that is fatally flawed, conceptually and logically.’

In this paper I explain that the conceptual and logical flaws in the fiscal theory of the price level that attract Buiter’s ire are an inevitable consequence of the attempt to

apply frictionless models to questions of monetary theory. Frictionless models of money and the price level simply do not make sense.

Frictionless models of money do not make sense because they preclude by construction all the properties of money and have no role for nominal values or the price level. The concept of a price level is not relevant in a world where only real relative prices exist and there is no medium of exchange whose purchasing power is of concern to rational agents. For the same reason there is no role for credit, banking or any role for the central bank. Without the existence of money as means of final settlement the concept of credit is also not adequately defined as the possibility of default is eliminated. Banks and central banks have nothing to do in frictionless models. The absence of all these features of a monetary economy means that frictionless models are incapable of producing any theory of the price level – be it a fiscal or quantity theory. It therefore makes no sense to use real frictionless models to produce *theories* of the price level or to derive nominal or real interest rate rules that would be of relevance to a central bank.

The most startling property of frictionless models is the implication that money is a ‘friction’. The term ‘monetary friction’ is an oxymoron. How can money, which everyone knows is an ‘invention’ that overcomes trade and production frictions, be described as a friction? In reality, money overcomes trade frictions but in a frictionless Walrasian world where trade frictions are eliminated by the Walrasian or time-0 auction, imposing a role for money converts money into a friction – contra economic theory, history and common sense. Consequently it is not possible to include both money and a Walrasian auction in a model without producing conceptual

and logical flaws of the sort noted by Buiter (1999, 2002, 2004, 2005, 2007), McCallum (2003, 2004), Wallace (2001), Goodhart (2004, 2005) and others. Failure to recognise this has left monetary theory in the 20th century littered with conceptual confusion and threatens to do the same for the 21st century!

Frictionless general equilibrium theory is what it has always been, a real theory that has nothing to say about money, credit and banking. It is therefore a mistake to believe either that frictionless models are a good approximation to modern electronic payments systems, or that the world is converging on the properties of the well-specified Walrasian general equilibrium model as suggested by King (1999), Woodford (2003) or Cochrane (2005)¹. Frictionless general equilibrium models do not map anywhere into the world of money, credit and banking.

Consequently, in frictionless models of money and the price level, theorists are inevitably forced to conflate the concepts of nominal and *numeraire* prices and to redefine the price level as a form of relative price. But as Patinkin (1965) explained there is no theory of *numeraire* prices, they are of no theoretical significance, and in frictionless models the *numeraire* can be anything, even something that doesn't exist. To highlight the logical traps into which this leads the unwary Buiter (2002) suggested that the *numeraire* be phlogiston, the mythical substance one thought to cause combustion.

¹ King (1999, p. 48) asks: "Is it possible that advances in technology will mean the arbitrary assumptions necessary to introduce money into rigorous models will become redundant, and that the world may come to resemble a pure exchange economy? Electronic transactions in real time hold out that possibility." This conjecture is simply unfounded. The evolution of electronic money does not mean that the world is demonetizing or converging to the properties of a Walrasian general equilibrium system.

In this paper Cochrane's (2005) defence of the fiscal theory of the price level in a frictionless Walrasian general equilibrium model is used to demonstrate why frictionless models of money and the price level do not make sense. Similar arguments apply *mutatis mutandis* to Woodford's model and are presented in Rogers (2006, 2007).

To make the case that frictionless models of money and the price level simply do not make sense the rest of the paper is structured as follows. Section 1 provides a brief overview of the properties of frictionless models and the conceptual and logical flaws that arise from attempts to incorporate money and theories of the price level in such models. Frictionless models are what Hahn (1973a) called 'inessential' monetary economies. Such models have the property that the real frictionless core remains invariant to any 'monetary' extensions. Frictionless 'inessential' monetary models are seen to be the source of all the conceptual and logical flaws noted in the literature. Section 2 briefly outlines Cochrane's frictionless well-specified Walrasian general equilibrium model and its use in defence of the fiscal theory of the price level in world where money is Microsoft stock. Section 3 outlines the conceptual flaws in Cochrane's analysis. The Euler equation is shown to be the only well-specified Walrasian general equilibrium component in Cochrane's model. All the other elements are shown to be 'inessential' additions in the sense of Hahn (1973a). Cochrane's frictionless model is nothing more than model of efficient or perfect barter-what some today call 'perfect record keeping' - and is thus unable to reveal anything about nominal prices or theories of the price level or shed any light on the foundations of monetary theory. Section 4 concludes with some thoughts on where the foundations of monetary theory may be found.

1 Properties of frictionless models as the source of conceptual dissonance

Cochrane's (2005, pp. 501-502, emphasis added) vision of a frictionless world where stock can become money is presented by his opening conjecture:

'Assume Microsoft stock becomes *numeraire*, unit of account *and medium of exchange*. When you buy coffee, you deliver a fraction of a Microsoft share, or a banknote check or electronic transfer that promises such payment. Bonds promise future payment of a share of Microsoft stock. Clearly, such a monetary system can establish a well-defined price level.'

The analysis presented below will explain why this vision is a mirage. The frictionless model that Cochrane uses to support this vision is said to be a well-specified Walrasian general equilibrium system but it is well known that such models have *no role for money or any medium of exchange* be it equity or anything else. Furthermore, frictionless models have no role for nominal values or the concept of the price level – only real relative prices matter to agents in a frictionless well-specified Walrasian general equilibrium model.

Nevertheless, the *raison d'être* for employing the Walrasian general equilibrium model is explained by Cochrane (2005, p. 503-504, emphasis added) in the following terms:

'Throughout economics, *frictionless competitive models are the benchmark, the foundation upon which we add interesting frictions*. Yet monetary economics has so far crucially relied on a *big friction at the short end of the yield curve* in order to even start talking about the price level.... The main contribution of this paper is to address these and related *theoretical criticisms of the fiscal theory*. The theory can work in a perfectly standard and *well-specified Walrasian economic model*, one in which the *government has no special status* and one in which all budget constraints are satisfied at both equilibrium and disequilibrium price levels. It may or may not apply to a given time and place but *it is at least a theoretically coherent possibility*'.

Here Cochrane is proposing to determine the price level in a frictionless well-specified Walrasian general equilibrium model without the 'big friction', i.e. the use

of money imposed by a cash-in-advance (CIA) constraint. Notice that, not only has money become a friction rather than a lubricant, but Cochrane is also claiming he can determine the price level in a well-specified Walrasian general equilibrium model without the quantity equation or CIA constraint. In this paper I also show that this is *not* a theoretically coherent possibility.

The fundamental flaw in frictionless models of money is this: the auction necessary to render the models frictionless precludes any useful role for money. The auction is the familiar Walrasian or time-0 auction described by Lucas (1984) or Ljungqvist and Sargent (2004)². Ljungqvist and Sargent (2004, p. 217, emphasis added) define the Walrasian or time-0 auction as follows:

“In the competitive equilibrium, all trades occur at $t = 0$ in one market. Deliveries occur later than $t = 0$, but no more trades. A vast clearing *or credit system operates at $t = 0$. It ensures that condition (8.5.1) [the household’s budget constraint] holds for each household i . A symptom of the once-and-for-all trading arrangement is that each household faces one budget constraint that accounts for all trades across dates and histories.”*

Hahn (1965, 1973, 1982) has consistently drawn attention to the consequences of employing such an auction. In particular, Hahn (1982, p. 1, emphasis added) explains why a well-specified Walrasian general equilibrium or Arrow-Debreu model has no role for money under such an auction:

The most serious challenge that the existence of money poses to the theorist is this: the best developed model of the economy cannot find room for it. The best-developed model is, of course, the Arrow-Debreu version of a Walrasian general equilibrium. *A world in which all conceivable contingent future contracts are possible neither needs*

² Lucas (1984, p. 10, emphasis added) describes the Walrasian auction in these terms:

‘A central feature of this model is that all trading occurs in a central market, with all agents present. In such a setting, the position of each agent is fully described by a single number: his wealth, or the market value of all the claims he owns (endowment). The command of any one claim over goods is fully described by its market value, which is to say *all claims are equally liquid*.’

nor wants intrinsically worthless money. . The point is obvious and has been made quite often. But it is doubtful that it has been fully taken on board.'

In short, this means that the functions of money are eliminated by construction in frictionless models. In frictionless models the assumptions necessary for the construction of the model are effectively substitutes for money as suggested by Laidler (1990). As Goodhart (2004, 2005) has also explained, associated properties of the time-0 auction such as the complete markets assumption, agents' single life-time budget constraint and the equivalence between the transversality and no bankruptcy conditions are all properties of frictionless models that effectively eliminate any role for money, credit and banks.

The correct interpretation of frictionless models as non-monetary models has been provided by McCallum (2003, pp. 1-2) when he describes 'trading' in a Walrasian system as:

'..an accounting system of exchange is one in which there is no money but exchanges are conducted by means of signals to an accounting network, with debits and credits to the wealth accounts of buyers and sellers being effected with each exchange. In the present paper, as in McCallum (1985), I will classify the latter type of system as non-monetary. In effect, an accounting system of exchange is a highly efficient form of barter'.

This form of efficient barter is completely different from real world barter which is characterised by frictions such as the double coincidence of wants. McCallum's highly efficient form of barter is in effect perfect barter in a frictionless world based on a time-0 auction.

Perhaps Hahn's and McCallum's advice has not been fully taken on board by exponents of frictionless models because such models possess another subtle but subversive property –they are what Hahn (1973a) called 'inessential' monetary

economies. An ‘inessential’ monetary economy in the sense of Hahn (1973a, p. 230, emphasis added) is one where: ‘.....*money is inessential in the sense that no monetary variable need enter into the description, or determination, of that economy’s equilibrium.*’

Frictionless models such as Cochrane’s well-specified Walrasian general equilibrium system have all of these properties by construction. What seems to have been overlooked in the literature is the conceptual dissonance that is produced by the use of such frictionless ‘inessential’ monetary models. Conceptual problems arise because although appending money in some form leaves the core frictionless general equilibrium structure invariant there is no theoretical basis for appending a role for money. As Hahn (1973a, p.233 emphasis added) went on to conclude:

‘But the inessential economy does not need money and one must give reasons for grafting on to it *monetary constraints. These reasons have not been given.*’

In other words, the practice of attaching a quantity equation or a CIA constraint to a frictionless model has no basis in Walrasian general equilibrium theory.

Frictionless models are therefore what they have always been; accounting systems of exchange or models of ‘perfect record keeping’ that are effectively models of perfect barter. In such frictionless models nominal prices are not defined and the concept of a price level is redundant –it serves no useful purpose under a Walrasian or time-0 auction. *Numeraire* prices exist and can be defined in terms of anything as *numeraire*, even something that doesn’t exist, like phlogiston. However, *numeraire* prices should not be confused with nominal prices and a price index constructed from *numeraire* prices as a measure of the general price level has no analytical relevance.

The fact that money can be appended in an ‘inessential’ fashion leaving the frictionless real core intact has however led to the long list of conceptual and logical anomalies noted in the literature. Examples illustrative of the nature of conceptual dissonance endemic to frictionless models include Patinkin’s (1965) attempt to insert money into the utility function, Clower’s (1967) attempt to impose the cash-in-advance constraint, Wallace’s (2004) demonstration that attempts to insert a central bank into a ‘cashless’ Arrow-Debreu economy produces conceptual puzzles and the mistaken belief that the evolution of e-money means that the world is somehow converging on the properties of the frictionless model. Consider each in turn.

Patinkin’s attempt to insert money-in-the-utility (MIU) function failed because in the context of a frictionless Walrasian general equilibrium model there is no point in distinguishing between the utility of money and the utility of all other commodities if all commodities are equally liquid under a Walrasian or time-0 auction. As money has no role in a model with a time-0 or Walrasian auction it cannot have utility different from that of any other commodity. Yet, in reality money clearly has utility different from that of other commodities because it is the universal medium of exchange. Patinkin’s failure to recognise that his frictionless Walrasian general equilibrium model effectively precluded by construction any special utility for money explains why his *Money, Interest, and Prices* failed to integrate monetary and value theory.

Clower (1967) exposed this weakness in Patinkin’s vision and attempted to remedy it by imposing a role for money as a medium of exchange or final settlement, by imposing a CIA constraint. Clower initially failed to see that by replacing Patinkin’s ‘inessential’ monetary extension to the frictionless Walrasian model –MIU - with his

own ‘inessential’ addition –the CIA constraint - he converted money from something that overcomes friction in reality into something that created friction in the model. However, as Clower (1984, p. 257) later realised, that was contrary to economic theory and common sense³. Cochrane’s model with the ‘big friction’ attached -the CIA constraint - exhibits exactly this property.

More recently, Wallace (2004) revealed that attempts to insert a central bank into a cashless Arrow-Debreu economy produced a string of insolvable conceptual puzzles. Clearly, it will not be possible to prove the existence of a competitive equilibrium in a model that contains a monopolist –which is effectively what a central bank is. A central bank is a price setter not a price taker. Hence Wallace demonstrates that it is not possible to find a role for a central bank in a frictionless or cashless well specified Walrasian general equilibrium model. Yet it is just such a frictionless or cashless world that Woodford (2003) and Cochrane (2005) propose as a *theoretical foundation* for monetary theory and policy⁴.

A more serious misconception underlying the use of frictionless models is the relatively widespread belief that the evolution of e-money means the economy is somehow converging on the properties of the frictionless Walrasian or Arrow-Debreu general equilibrium system. This is an idea suggested by King (1999), is endorsed by Woodford (2003, p. 31) and is also embraced by Cochrane (2005, p. 505) who

³Clower (1984, pp. 257) to his credit, realised that imposing a CIA constraint on a frictionless model meant that: ‘...the choice alternatives confronting households were more restrictive in a money than in a barter economy, which meant that monetary exchange is less efficient than barter exchange, contrary to both common sense and two hundred years of conventional wisdom. Something obviously was wrong. But what?’

⁴Recall that Lucas (1984) defends the CIA constraint on empirical not theoretical grounds while Woodford and Cochrane offer a theoretical defence of frictionless models. This paper deals with the latter question and the empirical relevance of frictionless models is not considered here although it would be legitimate to doubt the empirical relevance of frictionless models as does Goodhart (2004).

believes that his well-specified Walrasian general equilibrium model is a good approximation to the US Federal payments system. A similar misconception appears to be embraced by Wright (2005, p. 307) who associates the properties of the time-0 auction with the concept of perfect record keeping and conjectures that as the evolution of technology suggests the economy is converging on a state of perfect record keeping the need for a medium of exchange will disappear. These ideas are simply mistaken. No amount of improved record keeping via e-technology can induce an existing monetary economy to converge on the properties of a frictionless Arrow-Debreu world based on a time-0 auction. Hoover (1988 p. 97, emphasis added) who offered an elegant critique of earlier frictionless models, known at the time as the ‘New Monetary Economics’, explains why the evolution of e-money does not mean that the world is converging on the time-0 auction:

‘The fact that computerization may allow us to dispense with notes and coins, does not transform our economy from one in which transactions are made in a higgledy-piggledy uncontrolled manner into one in which they are coordinated by central auction.’

The technological progress that enables the use of e-money means only that the form of the medium of exchange is changing -*not that it is disappearing*. The evolution of e-money does not mean that the world is demonetizing – *contra* Woodford (1998). The world of e-money is not converging on the properties of the time-0 auction. Hence, as non-monetary models, frictionless models do not map anywhere into the world of e-money. Consequently the exponents of frictionless models find themselves on the horns of a dilemma. On the one hand, frictionless models are moneyless systems, but, on the other hand, adding the CIA constraint converts money into a friction, *contra* both economic theory and commonsense. Attempts to escape this dilemma produce the examples of conceptual dissonance outlined above.

The difficulty for Cochrane's vision of *money as Microsoft stock* should now be apparent. In Cochrane's *frictionless model* Microsoft stock may serve as a store of value but it has no role as a medium of exchange or final settlement - *contra* his opening conjecture that Microsoft stock could become the universal medium of exchange. In the frictionless model there is no medium of exchange. *In reality*, money, cash or e-transfer, will dominate Microsoft stock as the medium of exchange. Cochrane's idea that Microsoft stock can be used to pay for coffee is at best a non-operational thought experiment. In reality Microsoft stock is only one of a myriad of assets all with different liquidity characteristics and associated transactions costs. In reality money dominates all of these assets because of their inferior liquidity characteristics. The e-revolution does not mean that Microsoft or any other stock, will replace the dollar. Of course, coffee can now be purchased with an electronic purse but the transaction requires transfer of ownership of the medium of exchange –an electronic book-keeping entry –that transfers ownership of the dollar value where the dollar is both the *numeraire* and the medium of payment. That process extends all the way to the central bank in modern Real Time Gross Settlement (RTGS) systems. Such transactions are in no way related to the thought experiment personified by the time-0 auction.

With this brief overview of the properties and conceptual issues raised by frictionless models we are in a position to assess Cochrane's model. The description of the model will be kept to the minimum necessary to follow the argument. For additional detail the reader should consult Cochrane (2005) and Sargent (1987).

2 Cochrane's well-specified Walrasian general equilibrium model

Cochrane implicitly presents three nested versions of his Walrasian general equilibrium system: Version (i) is a Walrasian general equilibrium model onto which a CIA constraint or quantity equation has been grafted –it has the ‘big friction’ at the short end of the yield curve; version (ii) amends version (i) to allow households to exchange excess cash holdings for bonds overnight, but households are still subject to the CIA constraint on intraday trades; and version (iii) removes the CIA constraint on intraday trades to produce the frictionless or ‘completely cashless’ case. The medium of exchange function of money is eliminated in version (iii). It should be noted that Cochrane’s description of version (ii) is potentially misleading because he often refers to it as ‘cashless’ and ‘frictionless’ despite the existence of the CIA constraint. It is only version (iii) of the model that is completely cashless and hence frictionless in terms of Cochrane’s terminology. As it is with the relationship between the frictionless and ‘big friction’ versions of the model that we are primarily concerned, only versions (i) and (iii) will be discussed in this paper.

Version (i): Cochrane’s monetary model with the ‘big friction’ at the short end of the yield curve

Cochrane (2005) employs a Lucas tree model with a cash-in-advance constraint as presented by Sargent (1987, Table 5.1, p. 158) who provides a detailed outline of the trading pattern and the restrictions on households needed to justify a role for an intrinsically worthless medium of exchange in the model. Without these restrictions money as a medium of exchange would have zero exchange value (Hahn, 1965 and

Sargent 1987, chapters 3, 5).⁵ The model has four components – fruit producing trees, equities, bonds and money - in an intertemporal setting with trading days extending out to an infinite horizon. Three types of securities are traded in Sargent’s model, equities in trees, government issued currency and one period state contingent *claims to currency* –government bonds. But only currency and nominal government bonds appear in budget constraints as Cochrane (2005, p. 508) argues that equities in trees will be in zero net supply and consequently have no effect on the equilibrium solution; ‘...claims not provided by the government [i.e. equities issued by households] are in zero net supply and their presence or absence has no effect on the equilibrium prices or allocations’. Equities are an inside asset.

There are many *identical households* and a government. Each household owns a tree that produces fruit (dividends) but households are precluded from consuming their own fruit (to avoid an autarky solution). Identical households maximise a standard utility function, $E_0 \sum_{t=0}^{\infty} \mathbf{b}^t u(c_t)$, have shopper and worker characteristics, and enter period t with money balances M_{t-1} and one period *nominal* discount bonds with face value B_{t-1} . The government chooses a state-contingent sequence of one-period *nominal* debt, money and primary surpluses, $\{B_t^s, M_t^s, s_t\}$ and each are random variables. The model starts at $t = 0$ so B_{-1} and M_{-1} are fixed (they may both be set to zero).

⁵ As Sargent (1987, p. 136) explains for the case of a Lucas tree model: “In this economy, assets are valued according to the value of the stream of consumption that they support. An unbacked inconvertible currency promises to pay off nothing in the future. We have seen that introducing an asset with such a payoff stream into Lucas’s tree model leaves the equilibrium interest rates unaltered and causes the asset to receive a zero value”.

In the bond market the household sells bonds for currency, pays lump sum taxes, $p_t s_t$, buys new bonds, B_t and leaves with currency (money), M_t^d . In the goods market the household receives an endowment, e_t of the consumption good (fruit from the tree) and cannot consume its own endowment so must trade with other identical households. The household has worker and shopper characteristics. The *shopper* uses the money from the asset market, M_t^d to buy fruit, c_t subject to a cash-in-advance constraint (equation numbers follow Cochrane (2005),

$$p_t c_t \leq M_t^d v \quad (5)$$

where $v = \text{velocity}$ in a quantity equation and p_t is the price level. The *worker* sells the fruit endowment e_t for money and gets cash $p_t e_t$ in return. In the monetary state of the model, the worker and shopper ‘go home’ and eat the fruit, c_t . Also they must hold, overnight, any money balances not converted into fruit by shopping, the balance is, $M_t^d - p_t c_t$, plus the money balances earned by working and equal to $p_t e_t$. Hence we have:

$$M_t = M_t^d + p_t (e_t - c_t) \quad (6)$$

To derive the household’s budget constraints Cochrane notes that they can trade arbitrary contingent claims in the asset market and so the *nominal price* of a one period state contingent *bond* at time t is given by;

$$Q_t = p_t E_t \left(m_{t,t+1} \frac{1}{1 + p_{t+1}} \right) \quad (7)$$

The expression $m_{t,t+j}$ is a real stochastic discount factor or pricing kernel. As equities have no influence on the equilibrium solution only government bonds and money appear in the household's period-by-period budget constraint;

$$B_{t-1} + M_{t-1} + p_t(e_t - c_t) = Q_t B_t + M_t + p_t s_t \quad (9)$$

Prohibiting the households from issuing money (to prevent arbitrage against interest-bearing bonds) and applying a *transversality condition* produces what Cochrane calls the present value budget constraint:

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} (s_{t+j} + c_{t+j} - e_{t+j}) \quad (11)$$

In general equilibrium the goods, money and asset 'markets' are described by the following three equations:

$$\mathbf{b}^j \frac{u'(e_{t+j})}{u'(e_t)} = m_{t,t+j} \quad (12)$$

$$M_t v = p_t c_t = p_t e_t \quad (14)$$

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_t} \right) \right] \quad (15)$$

Cochrane (2005, p. 504) describes (15) as a valuation equation or market clearing condition and not a constraint. Those familiar with Patinkin's microfoundations and 'market experiments' will see that expressions (12), (14) and (15) can be used to generate market-clearing loci for the goods (fruit), money and bond markets in interest rate-price (r, p) space as illustrated in Patinkin (1965, 1989 edition, Figure XI-2, p. 259).

Version (iii): Cochrane's completely cashless and frictionless model

To eliminate all frictions Cochrane removes the CIA constraint, the 'big friction', to produce what he calls the *completely cashless* model. This model is a moneyless model and frictionless in the sense that all the constraints imposed on households to generate a non-zero exchange value for cash have been removed. In this case the medium of exchange function of money is eliminated altogether, money demand disappears and along with it any M in equation (15). The model is reduced to the two equations:

$$\mathbf{b}^j \frac{u'(e_{t+j})}{u'(e_t)} = m_{t,t+j}. \quad (17)$$

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j}) \quad (22)$$

In equation (22) the absence of M_{t-1} indicates that the model is completely cashless and hence frictionless –there is no medium of exchange. Cochrane then argues that expression (22) now serves to determine the 'price level' p_t in a frictionless Walrasian general equilibrium system, given B_{t-1} , $m_{t,t+1}$ and s_{t+j} . Expression (22) is a statement of the fiscal theory of the price level. The quantity theory supposedly no longer provides a theory of the price level because the 'big friction' at the short end of the yield curve has been eliminated.

3 Interpreting Cochrane's model

As outlined in section 2, Cochrane faces a dilemma. The frictionless model, version (iii) has no role for money while version (i) with the CIA constraint converts money into a friction, contra economic theory and common sense. There is no role for money and/or credit in Cochrane's frictionless model because it is a model based on a

time-0 auction or ‘perfect record keeping’ with complete markets and a transversality condition imposed on agents.

The first feature of the relationship between versions (i) and (iii) of Cochrane’s model apparent to the reader is the fact that the CIA constraint plays no essential role in equilibrium. Cochrane (2005, p. 513, emphasis added) describes this frictionless interpretation of his model in the following terms:

‘The cash in advance constraint plays no essential role in the equilibrium. ...For any equilibrium of the frictionless [i.e. ‘cashless’] model stated so far, the same equilibrium holds if we eliminate the cash constraint and ...eliminate intraday cash.’

The equilibrium to which Cochrane refers is, of course, the Euler equation - expressions (12) or (17) - which remains invariant across all versions of the model. The fact that the Euler equation remains invariant across all versions of Cochrane’s model reveals it to be what Hahn (1973a, b) called an ‘inessential monetary economy’. The CIA constraint or quantity equation can be added to the model but it is an *inessential addition in the sense that there is nothing we can say about the equilibrium of the model with the CIA constraint that cannot be said about the equilibrium without the constraint.*

The fact that the Euler equation in expression (17) remains invariant across all versions of the model indicates that it *is the only well-specified component of Cochrane’s Walrasian general equilibrium model.* As explained by Fisher (1907), what we now call the Euler equation, expression (17), represents intertemporal trading of consumption streams made possible by the existence of a *perfect market* in IOUs that determines the discount factor, $m_{t,t+j}$. Azariadis (1993) describes this feature of the model as ‘inside money’ although it is better described as ‘perfect record keeping’

or perhaps ‘perfect credit’ under the time-0 auction as the latter excludes any role for money. Clearly, the bonds described in equation (22) can no longer be claims to currency as originally specified in version (i) of the model because currency no longer exists in the frictionless state. What, then, is the interpretation of equation (22)?

To begin with, note that there is no basis for distinguishing between bonds and stocks in a frictionless world. Both could facilitate intertemporal trade but if they did so at different rates that would represent a ‘friction’ that should be eliminated by arbitrage from any frictionless model. Hence, bonds and equities must be perfect substitutes in Cochrane’s world of perfect competition and frictionless markets. The frictionless model cannot operate with differentiated assets so there is no point in distinguishing between bonds and equities. Hence equation (22) should be dropped from version (iii) of Cochrane’s model, its frictionless state, as the perfect intertemporal loan market requires only a single asset⁶. Consequently, identifying the bond market independently of the Euler equation is simply another example of an ‘inessential’ elaboration of the model. There is nothing we can say about the general equilibrium of the model represented by equations (17) and (22) that we could not say using just equation (17), the Euler equation.

The fact that bonds are an inessential addition to Cochrane’s frictionless model exposes further awkward questions about the role of the government agent in such models. Cochrane attributes no special status to this agent but the agent behaves differently by issuing bonds and imposing taxes. Why is she doing this and what is the relationship between this agent and the auctioneer running the time-0 auction? The

⁶ As Wallace (2001, p. 851) observes, it is necessary to preserve the perfect market for IOUs or credit in these models.

obvious answer is that the government agent plays no role in the time-0 auction. In short the government agent is a redundant extension of the model. The same applies to an agent labelled the 'central bank'. Such 'super agents' have no role to play in well-specified Walrasian models and their presence amounts to nothing more than window-dressing – a concession to naive realism. Hence the answer to the question reveals the government agent as another inessential addition to the model – the actions of such agents may simply create frictions, such as tax wedges, that should be removed if welfare is to be improved. If the intention is to use a competitive general equilibrium model then introducing super agents in addition to the auctioneer will inevitably generate frictions. Wallace (2004) formally illustrates that conclusion by showing how the attempt to introduce a price-setting central bank is obviously incompatible with the proof of existence of competitive equilibrium in an Arrow-Debreu model.

The correct interpretation of Cochrane's model therefore involves the Euler equation only: the quantity equation and the bond valuation equations are 'inessential' additions in the sense of Hahn (1973a, b). The Walrasian general equilibrium model presented by Cochrane is therefore nothing more than a version of Fisher (1907). Technology is represented by the fruit trees, tastes by the utility function and the perfect market for loans is represented by the time-0 auction⁷. As Samuelson (1967) noted, Fisher's analysis is isomorphic with any Walrasian general equilibrium model of intertemporal trading using the perfectly competitive loan market or underlying Walrasian auction. Fisher's model is then easily generalised to accommodate additional commodities that we can label 'bonds' and 'money' which can be traded

⁷ Complete specification of Cochrane's model requires Fisher's triple equality and should include the marginal productivity of the fruit trees. As Wallace (2001, p. 851) points out, this explains why money-in-the-production function (MIP) suffers the same fate as MIU.

over time using the perfect market for loans as embodied in Cochrane's equities. Such models satisfy Walras's Law by construction but they have nothing to tell us about monetary theory.

To confirm this conclusion consider three additional features of Cochrane's analysis; (i) the implications of Walras's Law, (ii) the use of asset-pricing theory to determine the 'price level', and (iii) the interpretation of *numeraire* prices.

Walras's Law

At one point Cochrane (2005, p. 506 -the equation numbers are (3) and (4)) identifies what he perceives to be a problem with a model that includes both equations (14) and (22) on the grounds that he has two equations to solve for only one unknown, p . But Cochrane's interpretation of the model is incorrect. In a well-specified Walrasian general equilibrium system of n markets, Walras's Law tells us that only $n-1$ of these markets are independent and the same is true of any market-clearing loci derived from them. Consequently one of the markets can always be dropped when determining the equilibrium solution in a Walrasian model. Hence in Cochrane's 'big frictions' version of the model we have three markets, fruit, bonds and money but only two independent market-clearing loci. We can drop any one of these equations, say equation (12), the fruit market-clearing locus in (r,p) space, leaving just the money and bonds market-clearing loci. But if dropping the goods market is permitted by Walras's Law what theory of the price level do we have then? – the model contains both the QTPL and FRPL to determine the price level! The correct answer, of course, is that the model has nothing to say about the debate between the FTPL and QTPL. Well-specified Walrasian general equilibrium models don't have and don't need a theory of the price level. In this respect, the debate between the FTPL and QTPL

simply reflects the earlier futile debate between loanable funds (bond market) and liquidity preference (money market) theories of the rate of interest when conducted in the context of a well-specified Walrasian general equilibrium model.

Asset-pricing theory and the price level

The fact that the price level is redundant in Cochrane's (2005, p. 511) model is reinforced when we realise that he is applying asset-pricing theory to determine the price level. The concept of the price level is not consistent across the frictionless and 'big friction' states of the model.

In the frictionless state, Cochrane makes some simplifying assumptions to transform equations (17) and (22) to read:

$$m_{t,t+1} = \mathbf{b} \quad (17')$$

$$p_t = p = (1 - \mathbf{b}) \frac{B}{s} \quad (22')$$

He then claims that as the discount factor is constant, *nominal interest rates* are said *to be positive* and the *price level* is said to be positive and constant. So if true, this would amount to the determination of the price level in the perfectly cashless and frictionless Walrasian general equilibrium model. These claims are false.

First, there is no nominal interest rate embedded in the discount factor in (17') as no medium of exchange exists. Second, expression (22') obviously does not determine the price level traditionally defined as the purchasing power of a unit of the medium of exchange. A moment's reflection will reveal that equation (22') is simply a version of the asset pricing formula of the type discussed by Sargent (1987, p. 96) so a new

variable, q_t -the fruit value of equity- should be introduced here. That is, for the simple utility function $u(c) = \ln c$, Sargent's asset pricing formula becomes:

$$q_t = \frac{\mathbf{b}}{1 - \mathbf{b}} d_t$$

Where q_t is defined as the *real asset price (measured in terms of the consumption good, fruit)* and d are the dividends, measured in fruit from the trees, paid to holders of equities.

Expression (22') is simply another special case of this form of solution represented by Sargent. The Lucas tree model without a CIA constraint is an asset-pricing model that determines the real price of equity measured in terms of fruit. Consequently, the LHS variable in Cochrane's equation (22') should be q_t - the fruit exchange value of equity in the trees and is clearly not what is traditionally understood by the price level- a measure of the purchasing power of a unit of the medium of exchange. Recall that equities are not the medium of exchange in a frictionless model –the model has no need for any such function of money.

By contrast, the variable p_t in equation (14) from the 'big frictions' state of the model, is *the purchasing power of a unit of the medium of exchange, M*. The definition of q_t in (22') is obviously not consistent with the definition of p_t in equation (14) as the medium of exchange has been eliminated from the frictionless model. Hence the definition of the price level is not consistent across the two states of the model. In the frictionless state we have a relative price, the fruit value of equity parading as the price level which is traditionally measured as the purchasing power of

a unit of the medium of exchange in the version of the model with the ‘big friction – the CIA constraint.’⁸

Numeraire prices

Finally, to be clear that a price level has no role in a Walrasian general equilibrium model (the model has no use for it), that claim should not be confused with the use of a *numeraire* in which to express prices. *Numeraire* prices are not nominal prices if the medium of exchange is not also the *numeraire*. In a Walrasian general equilibrium model there is no restriction on which commodity acts as the *numeraire* and even a non-existent ‘entity’ may act as *numeraire*. As noted previously, Buiter (2002) has suggested phlogiston, the mystical substance once thought to cause combustion so as to drive home the point that, in Walrasian general equilibrium models, the *numeraire* need not exist. Obviously if the *numeraire* does not exist it cannot be used as the medium of exchange. In Cochrane’s three-commodity model, prices may be expressed in terms of the quantity of fruit, bonds or equities. As fruit is the only consumption good it seems to be the natural *numeraire* and the ‘prices’ of equities and bonds must be quoted in terms of the quantity of fruit into which they can be converted at the equilibrium price vector. Prices in a Walrasian general equilibrium model are relative prices measured in terms of *quantities of the numeraire commodity* and *numeraire* prices have no theoretical significance, Patinkin (1965, chapter 2).

Thus, as Buiter (2002, p. 31) explains:

‘Any two commodities priced in phlogiston (or any imaginary and non-existent numeraire) will have a well-determined *relative* price. Determining the price of phlogiston (the *numeraire*) when phlogiston does not exist except as a word, is an intellectual bridge too far.’

⁸ Hoover (1988) pointed to a similar conceptual oversight in Fama’s (1980) attempt to apply Walrasian general equilibrium theory to monetary economics.

This traditional interpretation of the Walrasian general equilibrium system therefore makes it clear that Cochrane is *unable to present any theory of the price level* in the frictionless version of the model. Put simply, Cochrane's frictionless model has no role for the price level –the model determines only intertemporal commodity relative prices –how much fruit must be given up today to obtain a unit of fruit tomorrow or their implicit commodity interest rates (a fruit rate in Cochrane's model). *The price level is a redundant concept in such a model.* The price level is a concept that is relevant only to a world with a medium of exchange where its purchasing power is relevant to agents using the medium. There is no such *essential* relationship in a Walrasian general equilibrium model. Therefore, it is simply impossible to adjudicate between the monetary and fiscal theories of the price level in a Walrasian general equilibrium model. The model determines only relative commodity prices not a price level. Thus Cochrane's claim to determine the price level in a frictionless well-specified Walrasian general equilibrium model is false.

4 Conclusion

Both Cochrane and Woodford claim to present theories of the price level in frictionless models from which the medium of exchange has been eliminated. This claim is undoubtedly false. Frictionless models are, as McCallum (1985) explained, best interpreted as non-monetary accounting systems of exchange. In such models anything can then be designated as *numeraire*. But it is well known that *numeraire* prices have no analytical significance; there is no economic theory of *numeraire* prices. Claiming that frictionless models offer a theory of the price level defined in terms of *numeraire* prices would be, as Buiter (2002) argues, an intellectual bridge too far. It is not economic theory. Buiter (2007) now labels it *numerairology*.

The arguments presented above point to the incompatibility between Walrasian general equilibrium theories based on the time-0 auction and money. Stripping out the medium of exchange function of money as is proposed by exponents of frictionless models eliminates any useful analysis of money in the economic system. At best it could be argued that the time-0 auction is a substitute for money and that the real relative prices generated by the Walrasian general equilibrium system reflect the use of money. In that case money and the time-0 auction are substitutes as suggested by Laidler (1990). But that rather limits the use to which the Walrasian general equilibrium model can be put. It would not seem to be of much use for the analysis of monetary theory or policy. How then to proceed? To answer this question, consider Wallace's (2001) analysis of the future prospects for monetary theory and the role of search theory.

Wallace (2001) distinguishes between two categories of models; (i) models where money must meet certain *a priori* characteristics, and (ii) models where short-cuts are taken to generate a demand for the object that can be controlled by the central bank. The latter types are described as 'money-is-productive (MIP) models'. The frictionless models discussed in this paper fall into the second category and the arguments apply to MIU and MIP versions. Wallace correctly suggests that models in the second category *involve hidden inconsistencies and are inconsistent with any model in the first category*. This paper has exposed some of those hidden inconsistencies. It also exposes the inconsistency between the two categories of model identified by Wallace. This inconsistency rests on the fact that models with a time-0 auction or perfect record keeping have no role for money as a medium of exchange.

Failure to recognise this means that all the ‘short cuts’ adopted in the literature produce the hidden and not so hidden inconsistencies outlined in this paper.

Search theory falls into Wallace’s first category and is clearly a step in the right direction as it is based explicitly on the essential role of money as a medium of exchange in overcoming trade and production frictions. But as Lagos and Wright (2005) acknowledge, search theoretic models currently make too many strong assumptions to be useful as a tool for analysis of monetary policy. Consequently in their proposal for a unified framework for monetary theory with some empirical application they propose a ‘new short cut’. The new short cut consists of grafting a search-theoretic analysis that gives money an essential role in overcoming trading frictions in a decentralized ‘market’ onto a centralized Walrasian ‘market’. It must be apparent from the arguments presented above that the Lagos and Wright ‘short cut’ will also involve inconsistencies that undermine the search for a unified framework for monetary theory and policy analysis along the lines they propose.

The lesson to be learnt here is that models that give money an essential role do not map anywhere into well-specified Walrasian general equilibrium models. Hahn’s description of the properties of Walrasian general equilibrium theory needs to be taken seriously as does Solow’s advice that ‘microfoundations’ should not be interpreted as isomorphic with Walrasian general equilibrium theory.

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